On Integrating Description Logics and Rules under Minimal Hypotheses

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Abstract. A central and much debated topic in the Knowledge Representation and Reasoning community is how to combine open-world with closed-world formalisms, such as Description Logics (DLs) with Logic Programming. We propose an approach to defining the semantics of hybrid theories, composed of a DL and a Normal Logic Program (NLP) parts, which employs standard open-world semantics for the former and Pinto and Pereira's Minimal Hypotheses semantics (MHs) for the latter. As opposed to the currently employed semantics for hybrid DL-NLP KBs based on Stable Model (SM) semantics, our hybrid semantics guarantees the existence of models for any hybrid DL-NLP theory with consistent DL fragment and consistent DL-NLP ensemble. Because MHs features beneficial theoretical properties, like relevance and cumulativity, existential query answering tasks may not need to consider the whole hybrid KB, as it is necessarily the case with current state-of-the-art approaches based on the SM semantics.

1 Introduction

Description Logics (DLs) are a family of knowledge representation formalisms that are decidable fragments of first-order logic [2], where decidability is ensured via several syntactic restrictions. These restrictions lead to problems when expressing some non-tree like relationships. Such relations can easily be expressed using logic programming rules. Nevertheless, rule-based formalisms have their own shortcomings because typically they do not allow reasoning with unbounded infinite domains and hence cannot be used in many scenarios where modeling incomplete information is required.

A hybrid knowledge base (KB) has two components: a DL-KB[3] and a Logic Program (LP). In this work we focus on the same direction as, say, [8] and present a new approach of integrating DLs with normal Logic Programs (NLPs). Unlike the SM [3] based approaches like [8, 6], in our approach, odd loops over negation[4] in the rule part of

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[3] DL-KBs are usually called ontologies in the Semantic Web community. In this paper, we use these two terms interchangeably.

[4] When two rules depend on each other we say they form a loop. When such a loop is formed through default negated literals (DNLs) in the bodies of rules, we dub it loop over negation (LON). When there is an even (odd) number of DNLs through which the LON is formed we dub it even (odd) loop over negation (ELON/OLON).

a hybrid KB are not treated as modeling errors and hence not every hybrid KB containing OLONs needs to be inconsistent. Approaches based on Well-Founded Semantics (WFS) like [5] are three-valued and handle OLONs via the third *undefined* truth value.

Example 1. The affordable car problem.

Consider an online recommendation system for selling vehicles. The knowledge of the car sales company is described by the following ontology and NLP rule:

$$\begin{aligned} \mathit{Vehicle} &\equiv \mathit{Car} \sqcup \mathit{Van} \sqcup \mathit{Truck} && (1)\\ \mathit{Car} &\equiv \mathit{ABS} \sqcup \mathit{Airbagged} \sqcup \mathit{Automatic} && (2)\\ \mathit{AffordableCar} &\equiv \mathit{Car} \sqcap \neg(\mathit{ABS} \sqcap \mathit{Airbagged} \sqcap \mathit{Automatic}) \sqcap \mathit{StandardSeats} && (3)\\ \mathit{LuxuryCar} &\equiv \mathit{Car} \sqcap \mathit{ABS} \sqcap \mathit{Airbagged} \sqcap \mathit{Automatic} \sqcap \mathit{LeatherSeats} && (4)\end{aligned}$$

$$\mathit{StandardSeats(C)} \leftarrow not\ \mathit{LeatherSeats(C)} \qquad (5)$$

Vehicles for sale are cars, vans, or trucks (Axiom (1)); all cars always come with at least one additional feature (Axiom (2)); an affordable car misses at least one these features (Axiom (3)) and has standard seats; luxury cars have all three features and special leather seats (Axiom (4)). By default, a car is sold with standard seats, unless it is explicitly demanded by the customer that the car must have leather seats – Rule (5).

Suppose now there is a customer who will be happy if she gets an affordable car c, and her preferences regarding car systems are given as in the following rules:

$$\mathit{Automatic(c)} \leftarrow not\ \mathit{ABS(c)} \quad (6) \qquad \mathit{ABS(c)} \leftarrow not\ \mathit{Airbagged(c)} \quad (7)$$

$$\mathit{Airbagged(c)} \leftarrow not\ \mathit{Automatic(c)} \quad (8) \qquad \mathit{Happy} \leftarrow \mathit{AffordableCar(c)} \quad (9)$$

We need to find an affordable car while satisfying her preferences. Using the stable models as the semantic basis for the NLP part leads to no solution because the SMs are unable to assign models to the OLON formed by the rules $(6), (7)$ and (8). However, such a system is easily realizable in our approach. $\Diamond$

LONs in NLPs can be used to represent alternative choices, not unlike SAT problems, and in these cases the existence of a solution is guaranteed as long as no Integrity Constraints[5] (ICs) are added to the program. The Closed World Assumption (CWA) principle associated with the not operator is intended to enforce a skeptical stance, i.e., holding minimal beliefs. Although with LPs with no LONs we can always apply the CWA, with LPs with LONs there are several alternative minimal sets of beliefs one can assume — in this case we no longer use the CWA, but instead a Alternative World Assumption (AWA). The approach taken by the Minimal Hypotheses (MH) semantics [9], upon which our current work is based, considers its models to be the consequences of (set-inclusion) minimally assumed hypotheses, where the assumable hypotheses come from the atoms of DNLs in LONs. In the example the NLP part is used to represent

[5] An IC is a special kind of logic rule where the head is $\perp$. ICs are not part of NLPs, but (non-Normal) LPs are unions of "normal" rules with ICs. This way, a generate-and-test problem can be modeled by a LP using the normal rules as generators of candidate solutions (the models), and using the ICs as filters to discard unsatisfying candidate solutions.

a customer's preferences which we want to satisfy in a 2-valued fashion: a SM-based approach provides no solution, whereas a MH-based one does.

2 Minimal Hypotheses-based Semantics for Hybrid DL-NLP KBs

Our semantics for hybrid DL-NLP KBs is based upon a guess-and-check declarative fixed-point definition, an approach not unlike that of SMs (which are fixed-points of the Gelfond-Lifschitz[3] operator and are also defined via a guess-and-check).

A hybrid DL-NLP KB is a pair $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ where $\mathcal{O}$ is a DL-KB and $\mathcal{P}$ is an NLP. $\Sigma_{\mathcal{O}}$ denotes the signature (the set of predicate symbols and constants occurring in) of $\mathcal{O}$, $\Sigma_{\mathcal{P}}$ denotes the signature of $\mathcal{P}$, and $\Sigma_{\mathcal{K}}$ denotes the common signature of $\mathcal{K}$ — $\Sigma_{\mathcal{K}} = \Sigma_{\mathcal{O}} \cap \Sigma_{\mathcal{P}}$. $\mathcal{AB}_{\Sigma}$ denotes the set of all possible atoms over signature Σ. Our semantics for $\mathcal{K}$ takes into account the semantics of both of its components $\mathcal{O}$ and $\mathcal{P}$, where we consider the MH semantics for $\mathcal{P}$. MHs allows for several alternative models for $\mathcal{P}$, and the $\mathcal{O}$ has several models, thus the hybrid $\mathcal{K}$ must have several hybrid models. The literals of a model of each of $\mathcal{O}$ and $\mathcal{P}$ must be used by the other to allow for the possible entailment of more consequences. Coherence is enforced: explicitly negated literals entailed from $\mathcal{O}$ imply their default negated shared $\Sigma_{\mathcal{K}}$ counterparts in $\mathcal{P}$.

Definition 2. MH-based semantics of hybrid KB.

Let $\mathcal{O}$ be a consistent DL theory and $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid DL-NLP KB. A pair (I, M) is an MH-based hybrid model of $\mathcal{K}$ iff

- M is an MH model of $\mathcal{P} \cup (I^+ \cap \mathcal{AB}_{\Sigma_{\mathcal{K}}})$ with
- $\{not\ B : \neg B \in I^- \wedge B \in \mathcal{AB}_{\Sigma_{\mathcal{K}}}\} \subseteq M^-$ (coherence) and
- $\big(\mathcal{O} \cup (M^+ \cap \mathcal{AB}_{\Sigma_{\mathcal{K}}}) \cup (\{\neg B : not\ B \in M^- \wedge B \in \mathcal{AB}_{\Sigma_{\mathcal{K}}}\})\big) \cup I$ is consistent,

where $M = M^+ \cup M^-$, $M^+ \subseteq \mathcal{AB}_{\Sigma_{\mathcal{P}}}$, $M^- = \{not\ B : B \in \mathcal{AB}_{\Sigma_{\mathcal{P}}} \setminus M^+\}$; and $I = I^+ \cup I^-$, $I^+ \subseteq \mathcal{AB}_{\Sigma_{\mathcal{O}}}$, $I^- = \{\neg B : B \in \mathcal{AB}_{\Sigma_{\mathcal{O}}} \setminus I^+\}$. We use the term hybrid model instead of MH-based hybrid model whenever it is obvious from the context. ♢

In words, we define the semantics as a coupling of two different semantics via a synchronizing "interface" of ground atoms. A work closely related thereto is that of the so-called multi-context systems (MCSs): a framework that allows for combining arbitrary monotonic and non-monotonic logics [1]. A Hybrid KB in our approach can be taken as a multi-context system with two contexts, an ontology context and a program context. See [7] for a detail comparison with existing approaches.

In Example 1, a hybrid model would be (I, M) with (abbreviating predicate names)

$$I = \{SS(c), \neg LS(c), Air(c), ABS(c), AC(c), \neg Aut(c), Car(c), \neg LC(c), Veh(c)\} \text{ and}$$
$$M = \{SS(c), not\ LS(c), Air(c), ABS(c), not\ Aut(c), H, AC(c)\}.$$

Non-monotonicity in the NLP part is naturally supported by our formalism.

3 Reasoning

Given a model (I, M) for $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ and an atom A, $(I, M) \models A$ iff

- $\mathcal{O} \cup (M^+ \cap \mathcal{AB}_{\Sigma_\mathcal{K}}) \cup (\{\neg B : \textit{not}\ B \in M^- \wedge B \in \mathcal{AB}_{\Sigma_\mathcal{K}}\}) \cup I \models A$ whenever $A \in \mathcal{AB}_{\Sigma_\mathcal{O}}$, and
- $A \in M$ whenever $A \in \mathcal{AB}_{\Sigma_\mathcal{P}}$

Two reasoning tasks are essential: *consistency* and *entailment*. $\mathcal{K}$ is *consistent* iff there is at least one MH-based model for $\mathcal{K}$. For a given first-order atom A we say A is *credulously/skeptically entailed from* $\mathcal{K}$ (written as $\mathcal{K} \models_C A/\mathcal{K} \models A$) iff for some/every MH-based hybrid model (I, M) of $\mathcal{K}$ we have $(I, M) \models A$. The rules never violate the DL-safety restriction as the only way for $\mathcal{O}$ and $\mathcal{P}$ to communicate is via a finite set of shared ground atoms. Hence, the DL-safety restriction is trivially satisfied for all the rules. It follows from Def. 2 that both the consistency and entailment problems require guessing sets I and M such that the conditions imposed by the definitions are satisfied. In [7] we have shown that the complexity of these problems highly depends on the DL in the which the ontology part of the hybrid DL-NLP KB is formulated. E.g., for $\mathcal{SROIQ}$ [4] we get:

Theorem 3. *Complexity of the entailment and consistency problems.*

The consistency and the entailment problems in a hybrid DL-NLP KB are both N2EXPTIME-*complete.*

For the proof, we refer to [7] where we also provide a straight forward method for checking the entailment of an atom from a hybrid DL-NLP KB.

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