Spatial Concepts - a Rule Exploration

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Abstract

We explore everyday life concepts of spatial cognition. These are formalized and the rules holding between them are studied using the technique of Rule Exploration originating from Formal Concept Analysis. We determine a rule base for the chosen concepts.

1 Introduction

Concepts of spatial cognition, which are also called *spatial universals* [Br99], express how objects can be situated to an observer or to each other. They are represented by words like 'left', 'above' or 'inside'. Humans are able to infere new information from given facts expressed by such spatial concepts. For example, knowing that a certain object A is left to object B and B itself is left to a third object C, it is a trivial consequence that A must be left to C. It was our aim to determine all such rules holding for three objects with respect to a fixed set of well defined spatial concepts.

2 Definitions

Our objects will be convexe figures (i.e., compact convex three-dimensional point sets). Propositions like 'A is left to B' can be expressed in terms of first order predicate logic using a binary predicate *left* as follows: $left(A, B)$. Such a simple proposition is called an atom. In this way spatial concepts can be expressed by n-ary (mostly binary) predicates. We define predicates as follows:

predicate	definition	inverse
$equals(M,N)$	$M = N$	*equals*
$apart(M,N)$	$M \cap N = \emptyset$	*apart*
$touches(M,N)$	$M \cap N = bd(M) \cap bd(N) \neq \emptyset$	*touches*
$overlaps(M,N)$	$int(M) \cap int(N) \neq \emptyset, M \not\subseteq N, N \not\subseteq M$	*overlaps*
$intouches(M,N)$	$M \subset N, bd(M) \cap bd(N) \neq \emptyset$	*getstouched*
$inside(M,N)$	$M \subset N, bd(M) \cap bd(N) = \emptyset$	*includes*
$left(M,N)$	$\forall(x_m, y_m, z_m) \in M, (x_n, y_n, z_n) \in N : x_m \leq x_n$	*right*
$above(M,N)$	$\forall(x_m, y_m, z_m) \in M, (x_n, y_n, z_n) \in N : z_m \geq z_n$	*below*
$infront(M,N)$	$\forall(x_m, y_m, z_m) \in M, (x_n, y_n, z_n) \in N : y_m \leq y_n$	*behind*
$between(L,M,N)$	$conv(M \cup N) \cap int(L) \neq \emptyset$	

bd = boundary, int = interior, $conv$ = convex closure

Moreover, rules can be expressed as Horn clauses which are simple implications like $left(A,B) \wedge left(B,C) \rightarrow left(A,C)$ being true for every combination of any objects A,B,C described above. A set of Horn clauses will be called implicational base, if it is *complete* (i.e., all rules holding in the described part of reality can be infered from it) and *reduced* (i.e., none of the clauses can be infered from the other ones).

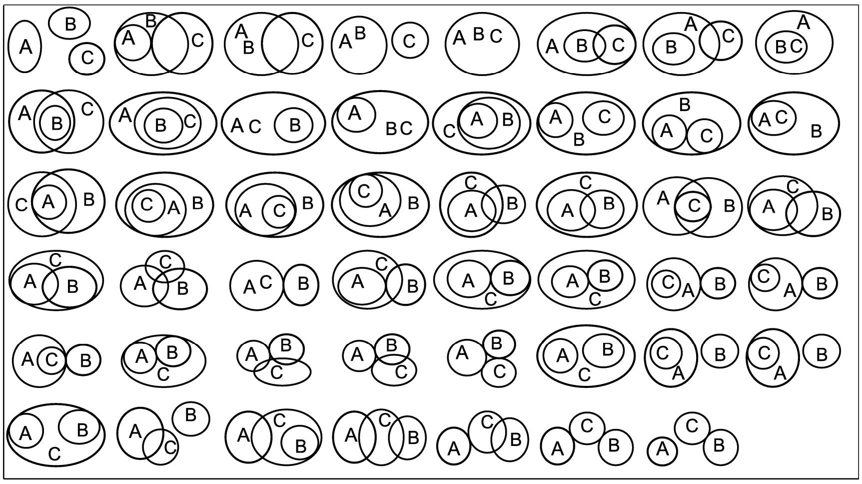
3 Preparations for Rule Exploration

Due to the amount of predicates, the exploration process exceeds the disposable time. Thus we split the set of predicates into two groups for two separated rule explorations. For the same reason we restrict the

exploration to rules involving only three objects (denoted by A,B, and C).
For the Rule Exploration algorithm only semantically different atoms are of interest. As one can easily see, *left*(A, B) and *right*(B, A) are semantically identical (i.e., they 'mean' the same). So we reduce the set of all (syntactic) possible atoms to a number of 45 semantic different atoms.
Furthermore we deal with some 'symmetry' properties. Obviously $left(A, B) \wedge left(B, C) \rightarrow left(A, C)$ is true. From this we can infere that $right(A, B) \wedge right(B, C) \rightarrow right(A, C)$ is also true as well as e.g. $above(A, B) \wedge above(B, C) \rightarrow above(A, C)$. Generally there are some basic operations which - exchanging some predicates of a Horn clause - preserve its truth value. Naturally, also 'relabeling' the variables of a Horn clause does not change its truth value. So from $left(A, B) \wedge left(B, C) \rightarrow left(A, C)$ follows $left(B, A) \wedge left(A, C) \rightarrow left(B, C)$ automatically by swapping A and B. Altogether, composing all those mentioned operations yields us a set of 288 permutations.
Finally there are some known trivial rules, which we enter in advance in order to shorten the actual exploration process. For example, the *left*- and the *right*-predicate are mutually exclusive, which means if *left*(A, B) is true for certain A and B then *right*(A, B) cannot also be true and vice versa. This we express by the Horn clause $left(A, B) \wedge right(A, B) \rightarrow \perp$, where '$\perp$' can be interpreted as 'false' or 'impossible'. We add these implications resulting from mutual exclusivity to the starting information.

4 Rule Exploration

The Rule Exploration algorithm interactively determines an implicational base of a defined 'universe' [Zi93]. Starting with the known information (atoms, symmetries and predefined rules as mentioned in the preceding section) it computes a 'hypothetical rule', presents it to an 'expert' (which knows the universe completely) and asks whether this is a real rule. If the expert confirmes this, the rule will be added to the implicational base, otherwise the expert will be asked to enter a counterexample. In this way the process proceeds with the next hypothesis until a complete implicational base is found.
As already mentioned we execute two rule explorations. The first one refers to the so called set-like predicates. The picture below shows all entered counterexamples followed by the computed implicational base.



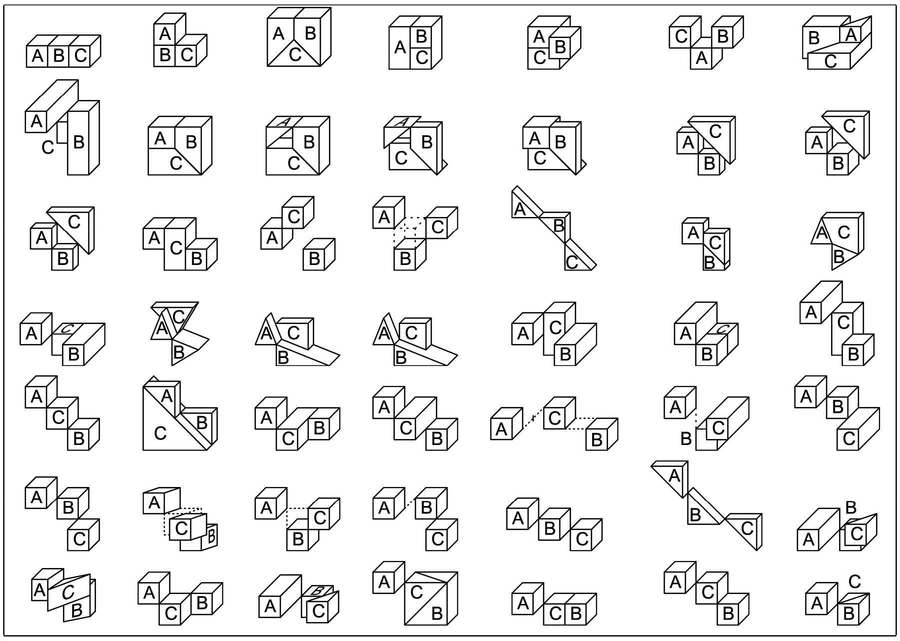
$apart(A,B), touches(A,B)$	$\rightarrow \perp$	$overlaps(A,B), intouches(A,B)$	$\rightarrow \perp$
$apart(A,B), overlaps(A,B)$	$\rightarrow \perp$	$overlaps(A,B), includes(A,B)$	$\rightarrow \perp$
$apart(A,B), intouches(A,B)$	$\rightarrow \perp$	$overlaps(A,B), equals(A,B)$	$\rightarrow \perp$
$apart(A,B), includes(A,B)$	$\rightarrow \perp$	$intouches(A,B), intouches(B,A)$	$\rightarrow \perp$
$apart(A,B), equals(A,B)$	$\rightarrow \perp$	$intouches(A,B), includes(A,B)$	$\rightarrow \perp$
$touches(A,B), overlaps(A,B)$	$\rightarrow \perp$	$intouches(A,B), includes(B,A)$	$\rightarrow \perp$
$touches(A,B), intouches(A,B)$	$\rightarrow \perp$	$intouches(A,B), equals(A,B)$	$\rightarrow \perp$
$touches(A,B), includes(A,B)$	$\rightarrow \perp$	$includes(A,B), includes(B,A)$	$\rightarrow \perp$
$touches(A,B), equals(A,B)$	$\rightarrow \perp$	$includes(A,B), equals(A,B)$	$\rightarrow \perp$

$$
\begin{array}{lcl}
equals(A,B), equals(A,C) & \rightarrow & equals(B,C) \\
includes(A,B), equals(B,C) & \rightarrow & includes(A,C) \\
includes(A,B), equals(A,C) & \rightarrow & includes(C,B) \\
intouches(A,B), equals(B,C) & \rightarrow & intouches(A,C) \\
intouches(A,B), equals(A,C) & \rightarrow & intouches(C,B) \\
overlaps(A,B), equals(A,C) & \rightarrow & overlaps(B,C) \\
touches(A,B), equals(A,C) & \rightarrow & touches(B,C) \\
apart(A,B), equals(A,C) & \rightarrow & apart(B,C)
\end{array}
$$

$$
\begin{array}{lcl}
includes(A,B), includes(C,A) & \rightarrow & includes(C,B) \\
intouches(A,B), includes(C,B) & \rightarrow & includes(C,A) \\
intouches(A,B), includes(A,C) & \rightarrow & includes(B,C) \\
intouches(A,B), intouches(C,A), intouches(B,C) & \rightarrow & \bot \\
overlaps(A,B), intouches(A,C), intouches(C,B) & \rightarrow & \bot \\
touches(A,B), includes(A,C) & \rightarrow & apart(B,C) \\
touches(A,B), intouches(C,A), intouches(C,B) & \rightarrow & \bot \\
touches(A,B), intouches(A,C), intouches(C,B) & \rightarrow & \bot \\
touches(A,B), overlaps(A,C), intouches(C,B) & \rightarrow & \bot \\
apart(A,B), includes(A,C) & \rightarrow & apart(B,C) \\
apart(A,B), intouches(C,A) & \rightarrow & apart(B,C)
\end{array}
$$

$$
\begin{array}{lcl}
equals(A,B) & \rightarrow & equals(B,A), \\
apart(A,B) & \rightarrow & apart(B,A), \\
touches(A,B) & \rightarrow & touches(B,A), \\
overlaps(A,B) & \rightarrow & overlaps(B,A), \\
intouches(A,B) & \rightarrow & getstouched(B,A), \\
getstouched(A,B) & \rightarrow & intouches(B,A), \\
inside(A,B) & \rightarrow & includes(B,A), \\
includes(A,B) & \rightarrow & inside(B,A)
\end{array}
$$

The second rule exploration deals with the coordinate-related predicates plus the *between* predicate. Below the entered counterexamples and the found implicational base are listed.



$$
\begin{array}{lcllcl}
left(A,B) & \rightarrow & right(B,A), & right(A,B) & \rightarrow & left(B,A) \\
above(A,B) & \rightarrow & below(B,A), & below(A,B) & \rightarrow & above(B,A) \\
infront(A,B) & \rightarrow & behind(B,A), & behind(A,B) & \rightarrow & infront(B,A) \\
between(A,B,C) & \rightarrow & between(A,C,B) & & &
\end{array}
$$

$$
\begin{array}{lcl}
left(A,B), left(B,A) & \rightarrow & \bot \\
above(A,B), above(B,A) & \rightarrow & \bot \\
infront(A,B), infront(B,A) & \rightarrow & \bot
\end{array}
$$

$$\begin{array}{lll}
\mathit{left}(A,B), \mathit{left}(C,A) & \rightarrow & \mathit{left}(C,B) \\
\mathit{above}(A,B), \mathit{above}(C,A) & \rightarrow & \mathit{above}(C,B) \\
\mathit{infront}(A,B), \mathit{infront}(C,A) & \rightarrow & \mathit{infront}(C,B) \\
\mathit{left}(A,B), \mathit{left}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot \\
\mathit{right}(A,B), \mathit{right}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot \\
\mathit{above}(A,B), \mathit{above}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot \\
\mathit{below}(A,B), \mathit{below}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot \\
\mathit{infront}(A,B), \mathit{infront}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot \\
\mathit{behind}(A,B), \mathit{behind}(A,C), \mathit{between}(A,B,C) & \rightarrow & \bot
\end{array}$$

We mention that the sets of counterexamples are not neccesarily reduced as the sets of implications are. They may contain redundant counterexamples.

5 Conclusion

Many implications of the obtained implicational bases seem to be trivial. This is not surprising since the predicates and their definitions represent concepts of man's everyday life. The advantage of the used technique is the guarantee of the implicational base's completeness. The information gained in that way could be used in AI or GIS systems for spatial reasoning. In the case of the first group of predicates the following transitivity table can be extracted very easily from the implicational base. It shows which predicates r_3 are possible for Ar_3B knowing that Ar_1B and Br_2C (omitting *equals* which is trivial):

Br_2C / Ar_1B	apart	touches	overlaps	intouches	getstou-ched	inside	includes
apart	a,t,o,n,g,s,c,e	a,t,o,n,s	a,t,o,n,s	a,t,o,n,s	a	a,t,o,n,s	a
touches	a,t,o,g,c	a,t,o,n,g,e	a,t,o,n,s	t,o,n,s	a,t	o,n,s	a
overlaps	a,t,o,g,c	a,t,o,g,c	a,t,o,n,g,s,c,e	o,n,s	a,t,o,g,c	o,n,s	a,t,o,g,c
intouches	a	a,t	a,t,o,n,s	n,s	a,t,o,n,s,e	s	a,t,o,g,c
getstou-ched	a,t,o,g	t,o,g,c	o,g,c	o,g,n,e	g,c	o,n,s	c
inside	a	a	a,t,o,n,s	s	a,t,o,n,s	s	a,t,o,n,g,s,c,e
includes	a,t,o,g,c	o,g,c	o,g,c	o,g,c	c	o,n,g,c,s,e	c

6 Perspectives

The Rule Exploration, which has proven to be very useful for this task, might also be used for related problems. For example a single implicational base could be computed including all predicates mentioned above using more ressources. Spatial reasoning involving the observer (with predicates like 'behind me') would be another subject of interest. It might even be extended to more than one observer, dealing with invariants and transformation rules. For the description of processes, spatial-temporal relations (like 'away from' or 'into') could be other rewarding objects of treatment.

References

[Al83] Allen J.F. (1983). 'Maintaining knowledge about temporal intervals' in *Communications of the ACM*, 26(11), 832-843.

[Br99] Brennan J. (1999). 'Spatial Universals as the Human Spatial Notion' in *Conceptional Structures: Standards and Practices*, Tepfenhart W., Cyre W. (Eds.), Lecture Notes in Artificial Intelligence 1640, Springer-Verlag, Berlin, 90-96.

[GW98] Ganter B., Wille R. (1998). *Formal Concept Analysis*, Springer-Verlag, Berlin Heidelberg.

[Zi93] Zickwolf M. (1993). *Rule Exploration - First Order Logic in Formal Concept Analysis*, TH Darmstadt, Preprint-Nr. 1580.