# Measuring Inconsistency for Description Logics Based on Paraconsistent Semantics *

Yue Ma[1,2], Guilin Qi[2], Pascal Hitzler[2], and Zuoquan Lin[1]

[1]Department of Information Science, Peking University, China
[2]AIFB, Universität Karlsruhe, Germany
{mayue,lz}@is.pku.edu.cn, {yum,gqi,hitzler}@aifb.uni-karlsruhe.de

**Abstract.** In this paper, we propose an approach for measuring inconsistency in inconsistent ontologies. We first define the degree of inconsistency of an inconsistent ontology using a four-valued semantics for the description logic $\mathcal{ALC}$. Then an ordering over inconsistent ontologies is given by considering their inconsistency degrees. Our measure of inconsistency can provide important information for inconsistency handling.

## 1 Introduction

Real knowledge bases and data for Semantic Web applications will rarely be perfect. They will be distributed and multi-authored. They will be engineered by more or less knowledgeable people and often be created automatically from raw data. They will be assembled from different sources and reused. Consequently, it is unreasonable to expect such realistic knowledge bases to be always logically consistent, and methods for the meaningful handling of such knowledge bases are being sought for.

Inconsistency has often been viewed as erroneous information in an ontology, but this is not necessarily the best perspective on the problem. The study of inconsistency handling in Artificial Intelligence indeed has a long tradition, and corresponding results are recently being transferred to description logics, which underly OWL.

There are mainly two classes of approaches to dealing with inconsistent ontologies. The first class of approaches is to circumvent the inconsistency problem by applying a non-standard reasoning method to obtain meaningful answers [1, 2] – i.e. to ignore the inconsistency in this manner. The second class of approaches to deal with logical contradictions is to resolve logical modeling errors whenever a logical problem is encountered [3, 4].

However, given an inconsistent ontology, it is not always clear which approach should be taken to deal with the inconsistency. Another problem is that when resolving inconsistency, there are often several alternative solutions and it would be helpful to have some extra information (such as an ordering on elements of the ontology) to decide which solution is the best one. It has been shown that analyzing inconsistency is

* We acknowledge support by the German Federal Ministry of Education and Research (BMBF) under the SmartWeb project (grant 01 IMD01 B), by the EU under the IST project NeOn (IST-2006-027595, http://www.neon-project.org/), by the Deutsche Forschungsgemeinschaft (DFG) in the ReaSem project, and by China Scholarship Council.

helpful to decide how to act on inconsistency [5], i.e. whether to ignore it or to resolve it. Furthermore, measuring inconsistency in a *knowledge base* in classical logic can provide some context information which can be used to resolve inconsistency [6–8].

There are mainly two classes of inconsistency measures in classical logic. The first class of measures is defined by the number of formulas which are responsible for an inconsistency, i.e. a knowledge base in propositional logic is more inconsistent if more logical formulas are required to produce the inconsistency [9]. The second class considers the propositions in the language which are affected by the inconsistency. In this case, a knowledge base in propositional logic is more inconsistent if more propositional variables are affected by the inconsistency [6, 10]. The approaches belonging to the second class are often based on some paraconsistent semantics because we can still find models for inconsistent knowledge bases in paraconsistent logics.

Most of the work on measuring inconsistency is concerned with knowledge bases in propositional logic. In [11], the authors generalized the work on measuring inconsistency in quasi-classical logic to the first-order case. However, it is not clear how their approach can be implemented because there is no existing work on implementing first-order quasi-classical logic.

At the same time, there are potential applications for inconsistency measures for ontologies, as they provide evidence for reliability of ontologies when an inconsistency occurs. In a scenario, where ontologies are merged and used together, such evidence can be utilised to guide systems in order to arrive at meaningful system responses.

In this paper, we propose an approach for measuring inconsistency in inconsistent ontologies. We first define the degree of inconsistency of an inconsistent ontology using a four-valued semantics for description logic $\mathcal{ALC}$. By analyzing the degree of inconsistency of an ontology, we can either resolve inconsistency if the degree is high (e.g. greater than 0.7) or ignore it otherwise. After that, an ordering over inconsistent ontologies is given by considering their inconsistency degrees. We then can consider those ontology which are less inconsistent and more reliable.

This paper is organized as follows. We first provide some basic notions in four-valued description logic $\mathcal{ALC}$ in Section 2. Our measure of inconsistency is then given in Section 3. Finally, we discuss related work and conclude the paper in Section 4.

## 2 Four-valued Models for $\mathcal{ALC}$

We assume that readers are familiar with the description logic $\mathcal{ALC}$ [12]. An $\mathcal{ALC}$ ontology is a pair $(\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is the set of class inclusions of the form $C \sqsubseteq D$ and $\mathcal{A}$ is the set of individual assertions in the forms $C(a)$ and $R(a, b)$. We use $\mathcal{L}_O$ to denote the set of all concepts, roles and individuals used in $O$, and assume the cardinality of $\mathcal{L}_O$ is finite, which is acceptable for applications.We review the four-valued semantics for $\mathcal{ALC}$ here, and see to [2] for details.

A four-valued interpretation (4-interpretation for short) is a pair $I = (\Delta^I, \cdot^I)$ with $\Delta^I$ as the domain, but $\cdot^I$ is a mapping which assigns to each concept $C$ a pair $\langle P, N\rangle$ of subsets of $\Delta^I$, and each role $R$ a pair $\langle R_P, R_N\rangle$ of subsets of $(\Delta^I)^2$, such that the conditions in Table 1 are satisfied, where proj$^+$ and proj$^-$ are functions defined

**Table 1.** Semantics of $\mathcal{ALC}4$ Concepts

| **Constructor Syntax** | **Semantics** |
|---|---|
| $A$ | $A^I = \langle P, N\rangle$, where $P, N \subseteq \Delta^I$ |
| $R$ | $R^I = \langle R_P, R_N\rangle$, where $R_P, R_N \subseteq \Delta^I \times \Delta^I$ |
| $o$ | $o^I \in \Delta^I$ |
| $\top$ | $\langle \Delta^I, \emptyset\rangle$ |
| $\bot$ | $\langle \emptyset, \Delta^I\rangle$ |
| $C_1 \sqcap C_2$ | $\langle \text{proj}^+(C_1^I) \cap \text{proj}^+(C_2^I), \text{proj}^-(C_1^I) \cup \text{proj}^-(C_2^I)\rangle$ |
| $C_1 \sqcup C_2$ | $\langle \text{proj}^+(C_1^I) \cup \text{proj}^+(C_2^I), \text{proj}^-(C_1^I) \cap \text{proj}^-(C_2^I)\rangle$ |
| $\neg C$ | $(\neg C)^I = \langle N, P\rangle$, if $C^I = \langle P, N\rangle$ |
| $\exists R.C$ | $\langle \{x \mid \exists y, (x, y) \in \text{proj}^+(R^I) \text{ and } y \in \text{proj}^+(C^I)\},$ $\{x \mid \forall y, (x, y) \in \text{proj}^+(R^I) \text{ implies } y \in \text{proj}^-(C^I)\}\rangle$ |
| $\forall R.C$ | $\langle \{x \mid \forall y, (x, y) \in \text{proj}^+(R^I) \text{ implies } y \in \text{proj}^+(C^I)\},$ $\{x \mid \exists y, (x, y) \in \text{proj}^+(R^I) \text{ and } y \in \text{proj}^-(C^I)\}\rangle$ |

as follows: $\text{proj}^+\langle P, N\rangle = P$, and $\text{proj}^+\langle R_P, R_N\rangle = R_P$; $\text{proj}^-\langle P, N\rangle = N$, and $\text{proj}^-\langle R_P, R_N\rangle = R_N$.

Intuitively, the first element $P$ (e.g. $\text{proj}^+(C^I)$) of the four-valued extension of a concept $C$ is the set of elements known to belong to the extension of $C$, while the second element $N$ (e.g. $\text{proj}^-(C^I)$) is the set of elements known to be not contained in the extension of $C$. The intuition is similar for the semantics of roles.In order to reason with inconsistency, we are free of these constrains, thus forming four epistemic states of the individual assertions under an interpretation: (1) we know the individual is contained, (2) we know the individual is not contained, (3) we have contradictory information, namely that the individual is both contained in the concept and not contained in the concept, (4)we have no knowledge whether or not the individual is contained. Next, we use the four truth values $\{t, f, \ddot{\top}, \ddot{\bot}\}$ from Belnap's four-valued logic [13], which denote truth, falsity, contradiction, and incompleteness, respectively, to distinguish them.

**Definition 1** *For instances $a, b \in \Delta^I$, concept name $C$, and role name $R$:*

$$
\begin{aligned}
C^I(a) = t, &\text{ iff } a^I \in \textit{proj}^+(C^I) \textit{ and } a^I \notin \textit{proj}^-(C^I),\\
C^I(a) = f, &\text{ iff } a^I \notin \textit{proj}^+(C^I) \textit{ and } a^I \in \textit{proj}^-(C^I),\\
C^I(a) = \ddot{\top}, &\text{ iff } a^I \in \textit{proj}^+(C^I) \textit{ and } a^I \in \textit{proj}^-(C^I),\\
C^I(a) = \ddot{\bot}, &\text{ iff } a^I \notin \textit{proj}^+(C^I) \textit{ and } a^I \notin \textit{proj}^-(C^I),\\
R^I(a, b) = t, &\text{ iff } (a^I, b^I) \in \textit{proj}^+(R^I) \textit{ and } (a^I, b^I) \notin \textit{proj}^-(R^I),\\
R^I(a, b) = f, &\text{ iff } (a^I, b^I) \notin \textit{proj}^+(R^I) \textit{ and } (a^I, b^I) \in \textit{proj}^-(R^I),\\
R^I(a, b) = \ddot{\top}, &\text{ iff } (a^I, b^I) \in \textit{proj}^+(R^I) \textit{ and } (a^I, b^I) \in \textit{proj}^-(R^I),\\
R^I(a, b) = \ddot{\bot}, &\text{ iff } (a^I, b^I) \notin \textit{proj}^+(R^I) \textit{ and } (a^I, b^I) \notin \textit{proj}^-(R^I).
\end{aligned}
$$

By the correspondence defined above, we can define a four-valued interpretation in terms of Table 1 or Definition 1. That is, the four-valued extension of a concept $C$ can be defined either as a pair of subsets of a domain or by claiming the the truth values for each $C^I(a)$, where $a \in \Delta^I$.

**Table 2.** Semantics of inclusion axioms in $\mathcal{ALC}4$

| Axiom Name | Syntax | Semantics |
|---|---|---|
| inclusion | $C_1 \mapsto C_2$ | $\Delta^I \setminus \text{proj}^-(C_1^I) \subseteq \text{proj}^+(C_2^I)$ |
| concept assertion | $C(a)$ | $a^I \in \text{proj}^+(C^I)$ |
| role assertion | $R(a,b)$ | $(a^I, b^I) \in \text{proj}^+(R^I)$ |

As to the semantics of inclusion axioms, it is formally defined in Table 2 (together with the semantics of concept assertions), which means that $C \sqsubseteq D$ is true under an interpretation $I$ if and only if for each individual which is not known to be not contained in the extension of $C$, it must be known to belong to the extension of $D$.

We say that a four-valued interpretation $I$ satisfies (a model of) an ontology $O$ iff $I$ satisfies each assertion and each inclusion axiom in $O$. An ontology $O$ is four-valued satisfiable (unsatisfiable) iff there exists (does not exist) a model for $O$. In this paper, we denote $\mathcal{M}4(O)$ as the set of four-valued models of an ontology $O$.

For an inconsistent ontology, it doesn't have classical two-valued models, but it may have four-valued models. For example, $I = \langle \{a\}, \cdot^I \rangle$, under which $A^I = \langle \{a\}, \{a\} \rangle$ is a model of the ontology whose $ABox = \{A(a), \neg A(a)\}$ and whose $TBox$ is empty. However, an ontology does not always have four-valued model if top and bottom concepts are both allowed as concept constructors. Take $\mathcal{T} = \{\top \sqsubseteq \bot\}$ for example. For any four-valued interpretation $I$, $\top^I = \langle \Delta^I, \emptyset \rangle$ and $\bot^I = \langle \emptyset, \Delta^I \rangle$. $\mathcal{T}$ has no four-valued model since $(\Delta^I \setminus \text{proj}^- \langle \top^I \rangle) = \Delta^I \not\subseteq \text{proj}^+ \langle \bot^I \rangle = \emptyset$, where $\Delta^I \neq \emptyset$ for DL interpretations. So in this paper we only consider the inconsistency measure of an inconsistent ontology which has no $\top$ and $\bot$ as concept constructors. Note that this assumption is relatively mild, as $\top$ can always be replaced by $A \sqcup \neg A$.

## 3 Inconsistency Measure

In this section, we use four-valued models of inconsistent $\mathcal{ALC}$ ontologies to measure their inconsistency degrees.

**Definition 2** *Let $I$ be a four-valued model of an ontology $O$ with domain $\Delta^I$,* the inconsistency set of $I$ for *$O$, written ConflictOnto$(I, O)$, is defined as follows:*

$$\textit{ConflictOnto}(I,O) = \textit{ConflictConcepts}(I,O) \cup \textit{ConflictRoles}(I,O),$$

*where ConflictConcepts$(I,O) = \{A(a) \mid A^I(a) = \ddot{\top}, A \in \mathcal{L}_O, a \in \Delta^I\}$, and ConflictRoles$(I,O) = \{R(a_1,a_2) \mid R^I(a_1,a_2) = \ddot{\top}, R \in \mathcal{L}_O, a_1, a_2 \in \Delta^I\}$*

Intuitively, ConflictOnto$(I, O)$ stands for the set of conflicting atomic individual assertions. To define the degree of inconsistency, we still need following concepts.

**Definition 3** *For the ontology $O$ and a 4-valued interpretation $I$,*

$$\textit{GroundOnto}(I,O) = \textit{GroundConcepts}(I,O) \cup \textit{GroundRoles}(I,O),$$

*where GroundConcepts*$(I, O) = \{A(a) \mid a \in \Delta^I, A \in \mathcal{L}_O\}$ *and GroundRoles*$(I, O) = \{R(a_1, a_2) \mid a_1, a_2 \in \Delta^I, R \in \mathcal{L}_O\}$

Intuitively, GroundOnto$(I, O)$ is the collection of different atomic individual assertions.

In order to define the degree of inconsistency, we use an assumption that only interpretations with finite domains are considered in this paper. This is reasonable in practical cases because only finite individuals can be represented or would be used. This is also reasonable from the theoretical aspect because $\mathcal{ALC}$ has finite model property — that is, if an ontology is consistent and within the expressivity of $\mathcal{ALC}$, then it has a classical model whose domain is finite.

**Definition 4** *The* inconsistency degree *of an ontology w.r.t. a model $I \in \mathcal{M}4(O)$, denote Inc$_I(O)$, is a value in $[0, 1]$ calculated in the following way:*

$$Inc_I(O) = \frac{|ConflictOnto(I, O)|}{|GroundOnto(I, O)|}$$

That is, The inconsistency degree of $O$ w.r.t. $I$ is the ratio of the number of conflicting atomic individual assertions divided by the amount of all possible atomic individual assertions of $O$ w.r.t. $I$. It measures to what extent a given ontology contains inconsistency w.r.t. $I$.

**Example 5** *Consider Ontology $O = (\mathcal{T}, \mathcal{A})$, where $\mathcal{T} = \{A \sqsubseteq B \sqcap \neg B\}, \mathcal{A} = \{A(a)\}$. A model of O is as follows: $I_1 = (\Delta^{I_1}, \cdot^{I_1})$, where $\Delta^{I_1} = \{a\}$, $A^{I_1}(a) = t$, and $B^{I_1}(a) = \ddot{\top}$. For this model, GroundOnto$(I_1, O) = \{A(a), B(a)\}$, and $B(a)$ is the unique element in ConflictOnto$(I_1, O)$. Therefore, Inc$_{I_1}(O) = \frac{1}{2}$.*

In [11], it has been shown that for a fixed domain, not all the models need to be considered to define an inconsistency measure because some of them may overestimate the degree of inconsistency. Let us go back to Example 5.

**Example 6** *(Example 5 Continued) Consider another model $I_2$ of O: $I_2 = (\Delta^{I_2}, \cdot^{I_2})$, where $\Delta^{I_2} = \{a\}$, $A^{I_2}(a) = \ddot{\top}$, $B^{I_2}(a) = \ddot{\top}$. $I_1$ and $I_2$ share a same domain. Since $|$ConflictOnto$(I_2, O)| = |\{B(a), A(a)\}| = 2$, we have $I_1 \leq_{Incons} I_2$ by Definition 7. This is because $\cdot^{I_2}$ assigns $contradiction$ to $A(a)$. However, $A(a)$ is not necessary a conflicting axiom in four-valued semantics. Therefore, we conclude that $Inc_{I_2}(O)$ overestimates the degree of inconsistency of O.*

We next define a partial ordering on $\mathcal{M}4(O)$ such that the minimal elements w.r.t. it are used to define the inconsistency measure for $O$.

**Definition 7** *(Model ordering w.r.t. inconsistency) Let $I_1$ and $I_2$ be two four-valued models of ontology O such that $|\Delta_1^I| = |\Delta_2^I|$, we say the inconsistency of $I_1$ is less than or equal to $I_2$, written $I_1 \leq_{Incons} I_2$, if and only if Inc$_{I_1}(O) \leq$ Inc$_{I_2}(O)$.*

The condition $|\Delta^{I_1}| = |\Delta^{I_2}|$ in this definition just reflects the attitude that only models with the same cardinality of domain are comparative. As usual, $I_1 <_{Incons} I_2$ denotes $I_1 \leq_{Incons} I_2$ and $I_2 \not\leq_{Incons} I_1$, and $I_1 \equiv_{Incons} I_2$ denotes $I_1 \leq_{Incons} I_2$ and $I_2 \leq_{Incons} I_1$. $I_1 \leq_{Incons} I_2$ means that $I_1$ is more consistent than $I_2$.

The model ordering w.r.t. inconsistency is used to define the preferred models.

**Definition 8** *Let $O$ be a DL-based ontology and $n(n \geq 1)$ be a given cardinality, the* preferred models w.r.t $\leq_{Incons}$ of size $n$*, written $PreferModel_n(O)$, are defined as follows:*

$$PreferModel_n(O) = \{I \mid |\Delta^I| = n; \forall I' \in \mathcal{M}4(O), |\Delta^{I'}| = n \textit{ implies } I \leq_{Incons} I'\}$$

That is, $\text{PreferModel}_n(O)$ is the set of models of size $n$ which are minimal w.r.t $\leq_{Incons}$.

The following theorem says that the cardinality of a domain is critical for measuring the inconsistency degree of an ontology, while the element differences can be ignored.

**Theorem 9** *Let $O$ be an ontology and $n(\geq 1)$ be any given positive integer. Suppose $I_1$ and $I_2$ are two four-valued models of $O$ such that $|\Delta^{I_1}| = |\Delta^{I_2}| = n$, $\{I_1, I_2\} \subseteq PreferModel_n(O)$. The following equation always holds:*

$$Inc_{I_1}(O) = Inc_{I_2}(O).$$

For simplicity of expression, we say an interpretation is *well-sized* if and only if the cardinality of its domain is equal to or greater than the number of individuals in $O$. Because of the unique name assumption of DL $\mathcal{ALC}$, an interpretation can be a model only if it is well-sized. Moreover, the following theorem asserts the existence of preferred models among the well-sized interpretations.

**Theorem 10** *For any given $\mathcal{ALC}$ ontology $O$ without concepts $\top$ and $\bot$ in the language, the preferred models among well-sized interpretations always exist.*

Above we consider the inconsistency degrees of an ontology w.r.t. its four-valued models, especially the preferred models. Now we define an integrated inconsistency degree of an ontology allowing for different domains.

**Definition 11** *Given an ontology $O$ and an arbitrary cardinality $n(n \geq 1)$, let $I_n$ be an arbitrary model in $PreferModels_n(O)$. The inconsistency degree sequence of $O$, say $OntoInc(O)$, is defined as $\langle r_1, r_2, ..., r_n, ...\rangle$, where $r_n = ModelInc(I_n, O)$ if $I_n$ is well-sized. Otherwise, let $r_n = *$. We use $*$ as a kind of null value.*

From theorem 9 and 10, the following property holds obviously.

**Proposition 12** *Assume $O$ is an inconsistent ontology and $OntoInc(O) = \langle r_1, r_2, ...\rangle$, and $N$ is the number of individuals of $O$, then*

$$r_i = \begin{cases} * & \textit{if } 0 < i < N, \\ r_i \neq * \textit{ and } r_i > 0 & \textit{if } i \geq N. \end{cases}$$

This proposition shows that for an ontology, its inconsistency measure cannot be a meaningless sequence — that is, each element is the null value $*$. Moreover, the non-zero values in the sequence starts just from the position which equals to the number of individuals in the ontology, and remains greater than zero in the latter positions of the sequence.As for an example, we first measure an extreme inconsistent ontology.

**Example 13** $O = \{C \sqcup \neg C \sqsubseteq C \sqcap \neg C\}$. *$O$ is inconsistent. Obviously, for any four-valued model $I = \langle \Delta^I, \cdot^I \rangle$ of $O$, $C$ is assigned to $\langle \Delta^I, \Delta^I \rangle$, so $OntoInc(O) = \{1, 1, ...\}$.*

After the inconsistency degree is defined for each ontology, we can use it to compare two ontologies one of which is less inconsistent. The ordering over inconsistent ontologies is defined as follows:

**Definition 14** *Given two ontologies $O_1$ and $O_2$. Suppose $OntoInc(O_1) = \langle r_1, r_2, ...\rangle$ and $OntoInc(O_2) = \langle r'_1, r'_2, ...\rangle$. We say $O_1$ is less inconsistent than $O_2$, written $O_1 \preceq_{Incons} O_2$, iff*

$$r_n \leq r'_n, \text{ for all } n \geq max\{k, k'\},$$

*where $k = min\{i : r_i \neq *\}, k' = min\{j : r'_j \neq *\}$.*

According to proposition 12, $\preceq_{Incons}$ is well-defined. In Definition 14, we compare the values from the position at which both sequences have non-null values because the null value $*$ cannot reflect useful information about the inconsistency of the ontology.

To compare two ontologies with respect to the ordering $\preceq_{Incons}$, by Definition 14, we have to compare two infinite sequences, which is practically very hard. When designing an algorithm to compare two ontologies, we can set a termination condition in order to guarantee that an answer will be obtained. Suppose time (resource) is used up and $\langle r_1, ..., r_n\rangle$ and $\langle r'_1, ..., r'_m\rangle$ are the already obtained partial sequences of OntoInc$(O)$ and OntoInc$(O')$, respectively. Then we can say that $O$ is approximatively less inconsistent than $O'$, denotes by $O \precsim_{Incons} O'$, if and only if $r_i \leq r'_i$ for all $1 \leq i \leq min\{m, n\}$.

**Example 15** *(Example 5 continued) Each preferred model $I$ of $O$ must satisfy that (1) it assigns one and only one individual assertion in $\{B(a), A(a)\}$ to contradictory truth value $\ddot{\top}$ — that is, $B^I(a) = \ddot{\top}$ and $A(a) = t$, or $B^I(a) = t$ and $A(a) = \ddot{\top}$; (2) it assigns other grounded assertions to truth values among the set $\{t, f, \ddot{\bot}\}$. So $|ConflictOnto(I, O)| = 1$. Consequently, $OntoInc(O) = \{\frac{1}{2}, \frac{1}{4}, ..., \frac{1}{2n}, ...\}$.*

*Suppose $O_1 = \{A \sqsubseteq B \sqcap \neg B, A \sqsubseteq C, A(a)\}$. In its preferred models, the individual assertions related to $C$ are not involved with contradictory truth value, so $OntoInc(O_1) = \{\frac{1}{3}, \frac{1}{6}, ..., \frac{1}{3n}, ...\}$. By definition 14, $O_1 \prec_{Incons} O$, which means that $O_1$ has less inconsistency percent than $O$ does.*

## 4 Related Work and Conclusion

To the best of our knowledge, this paper is the first work which can distinguish description logic based ontologies in many levels considering their different inconsistency degrees instead of only 0/1. In this paper, we mainly spells out for $\mathcal{ALC}$. The extension to more expressive languages is direct by extending four-valued semantics for them.

Our work is closely related to the work of inconsistency measuring given in [11], where Quasi-Classical models (QC logic [14]) are used as the underlying semantics. In this paper, we use four-valued models for description logics as the underlying semantics. This is because QC logic needs to translate each formula in the theory into prenex conjunctive normal form (PCNF). We claim that this is not practical, especially for a large ontology, because it may be quite time consuming and users probably do not like their ontologies to be modified syntactically. In this paper, we can see that four-valued models also provide us with a novel way to measure inconsistent degrees of ontologies.

It is also apparent that the inconsistency measure defined by our approach can be used to compute each axiom's contribution to inconsistency of a whole ontology by adapting the method proposed in [8], thereby providing important information for resolving inconsistency in an ontology.

In [11], every set of formulae definitely has at least one QC model because neither the constant predicate $t$ (tautology) nor the constant predicate $f$ (false) is contained in the language. However, corresponding to $t$ and $f$, the top concept $\top$ and bottom concept $\perp$ are two basic concept constructors for $\mathcal{ALC}$. This requirement leads to the possible nonexistence of four-valued models of an $\mathcal{ALC}$ ontology. Due to the space limitation, we presume that the ontologies do not use $\top$ and $\perp$ as concept constructors. The discussion for an arbitrary inconsistent ontology will be left as future work.

For practical implementations, we suggest that a termination condition be used. In the further work, we will work on some guidance on selecting such a termination condition and on how to measure the trade-off between early termination and accuracy.

For the implementation of our approach, we are currently working on the algorithm, which will be presented in a future paper.

## References


1. Huang, Z., van Harmelen, F., ten Teije, A.: Reasoning with inconsistent ontologies. In: Proc. of 19th International Joint Conference on Artificial Intelligence(IJCAI'05). Morgan Kaufmann (2005) 254–259
2. Ma, Y., Hitzler, P., Lin, Z.: Algorithms for paraconsistent reasoning with OWL. In: Proceedings of the 4th International Semantic Web Conference, ESWC07, Innsbruck, Austria, June 2007. (2007) To appear.
3. Schlobach, S.: Diagnosing terminologies. In: Proc. of AAAI'05. (2005) 670–675
4. Parsia, B., Sirin, E., Kalyanpur, A.: Debugging OWL ontologies. In: Proc. of WWW'05. (2005) 633–640
5. Hunter, A.: How to act on inconsistent news: Ignore, resolve, or reject. Data Knowl. Eng. **57** (2006) 221–239
6. Hunter, A.: Measuring inconsistency in knowledge via quasi-classical models. In: AAAI/IAAI. (2002) 68–73
7. Hunter, A., Konieczny, S.: Approaches to measuring inconsistent information. In Bertossi, L.E., Hunter, A., Schaub, T., eds.: Inconsistency Tolerance. Volume 3300 of Lecture Notes in Computer Science., Springer (2005) 191–236
8. Hunter, A., Konieczny, S.: Shapley inconsistency values. In Doherty, P., Mylopoulos, J., Welty, C.A., eds.: KR, AAAI Press (2006) 249–259
9. Knight, K.M.: Measuring inconsistency. Journal of Philosophical Logic **31** (2001) 77–98
10. Oller, C.A.: Measuring coherence using LP-models. J. Applied Logic **2** (2004) 451–455
11. Grant, J., Hunter, A.: Measuring inconsistency in knowledgebases. J. Intell. Inf. Syst. **27** (2006) 159–184
12. Baader, F., Calvanese, D., McGuinness, D., Nardi, D., Patel-Schneider, P., eds.: The Description Logic Handbook: Theory, Implementation, and Applications. Cambridge University Press (2003)
13. Belnap, N.D.: A useful four-valued logic. Modern uses of multiple-valued logics (1977) 7–73
14. Hunter, A.: A semantic tableau version of first-order quasi-classical logic. In Benferhat, S., Besnard, P., eds.: ECSQARU. Volume 2143 of Lecture Notes in Computer Science., Springer (2001) 544–555