Revisiting Semantics for Epistemic Extensions of Description Logics

Anees Mehdi and **Sebastian Rudolph**

{anees.mehdi,sebastian.rudolph}@kit.edu
Karlsruhe Institute of Technology, Germany

Abstract

Epistemic extensions of description logics (DLs) have been introduced several years ago in order to enhance expressivity and querying capabilities of these logics by knowledge base introspection. We argue that unintended effects occur when imposing the semantics traditionally employed on the very expressive DLs that underly the OWL 1 and OWL 2 standards. Consequently, we suggest a revised semantics that behaves more intuitively in these cases and coincides with the traditional semantics of less expressive DLs. Moreover, we introduce a way of answering epistemic queries to OWL knowledge bases by a reduction to standard OWL reasoning. We provide an implementation of our approach and present first evaluation results.

Introduction

In the early 80s, Hector J. Levesque argued for the need for a richer query language in knowledge formalisms (Levesque 1984). He advocated the idea of extending a querying language by the attribute *knows* denoted by $\mathbf{K}$ (also called *epistemic operator*, used akin to modalities in modal logics) thus enabling a sort of *knowledge base introspection* by making logical entailments of the knowledge base accessible from *within* the query language. Reiter (1992) makes a similar argument of in-adequacy of the standard first-order language for querying in the context of databases.

While propositional logic extended by epistemic operators has been widely studied and is well-understood, the introduction of $\mathbf{K}$ into first-order logic (as treated by Fitting and Mendelsohn 1998 and Braüner and Ghilardi 2006) brings about conceptual controversies concerning assumptions to be made about the domains of quantification, equality, (non-)rigidity of constants and the like.

Due to the extended reasoning capabilities, epistemic extensions have also been investigated (cf. e.g. (Donini et al. 1992; Donini, Nardi, and Rosati 1995; 1997; Donini et al. 1998)) in the context of Description Logics (DLs, cf. Baader et al. 2003), which recently have gained importance as the logical foundation of the OWL standard (the Web Ontology Language, cf. OWL Working Group 2009) that serves as one of the central technologies fueling the Semantic Web (Hitzler, Krötzsch, and Rudolph 2009).

Generally, epistemic DLs allow for introspection of the knowledge base by means of the epistemic operator $\mathbf{K}$ that can be applied to concepts and roles. The extension of the basic DL $\mathcal{ALC}$ (Schmidt-Schauß and Smolka 1991) by $\mathbf{K}$ called $\mathcal{ALCK}$, is presented by Donini et al. (1998), where a tableau algorithm for deciding the satisfiability problem is provided and the special task of answering queries in $\mathcal{ALCK}$ put to $\mathcal{ALC}$ knowledge bases is discussed.

To see the usefulness of the $\mathbf{K}$ operator for epistemic querying consider the following example. Assume we want to query for "known white wines that are not known to be produced in a French region" which can be solved by performing instance retrieval w.r.t. the epistemic DL concept

$$\mathbf{K}\mathit{WhiteWine} \sqcap \neg\exists\mathbf{K}\mathit{locatedIn}.\{\mathit{FrenchRegion}\}.$$

This query will not only retrieve the wines that are explicitly excluded from being French wines but also those for which there is just no evidence that they are French (neither directly nor indirectly via deduction). For the knowledge base containing

$$\mathit{WhiteWine}(\mathit{MountadamRiesling}) \quad \text{and}$$
$$\mathit{locatedIn}(\mathit{MountadamRiesling}, \mathit{AustralianRegion}),$$

the query would yield *MountadamRiesling* as a result, whereas the same query without epistemic operators would produce an empty result. Moreover, by adding additional information such as *MountadamRiesling* being located in a French region, the answer to the epistemic query would also become empty, which illustrates that introducing the epistemic operator into a logic brings about non-monotonicity.

Another typical use case for epistemic querying is integrity constraint checking: testing whether the axiom

$$\mathbf{K}\mathit{Wine} \sqsubseteq \exists\mathbf{K}\mathit{hasSugar}.\{\mathit{Dry}\} \sqcup \exists\mathbf{K}\mathit{hasSugar}.\{\mathit{OffDry}\} \sqcup \exists\mathbf{K}\mathit{hasSugar}.\{\mathit{Sweet}\}$$

is entailed allows to check whether for every named individual in the knowledge base that is known to be a wine it is also known (i.e. it can be logically derived from the ontology) what degree of sugar it has. Note that this cannot be taken for granted even if $\mathit{Wine} \sqsubseteq \exists\mathit{hasSugar}.\{\mathit{Dry}\} \sqcup \exists\mathit{hasSugar}.\{\mathit{OffDry}\} \sqcup \exists\mathit{hasSugar}.\{\mathit{Sweet}\}$ is stated in (or can be derived from) the ontology.

These examples illustrate an obvious added value of epistemic extensions of description logics in practical applications. However, epistemic operators (similar to other non-monotonic features) have not found their way into the OWL specification and current reasoners do not support this feature. Former research – focused on extending tableaux algorithms for less expressive languages – has not paced up with the development of reasoners for very expressive DLs. In fact, as we will discuss in the course of this paper, some expressive features like nominal concepts require special care when combined with the idea of introspection by epistemic operators.

This paper investigates the epistemic extension of the very expressive DL $\mathcal{SROIQ}$ (Horrocks, Kutz, and Sattler 2006), which serves as the logical basis of OWL 2 DL (OWL Working Group 2009), the most expressive member of the OWL family that is still decidable. When applying a semantics along the lines of Donini et al. (1998) to $\mathcal{SROIQ}$ we observe effects that clearly contradict natural requirements for epistemic reasoning (that we call backward compatibility).

This directly leads to the question for an altered semantics that "behaves well" also for $\mathcal{SROIQ}$. We introduce such a semantics and show that it complies with the proposed requirements.

With the more adequate semantics at hand, we then turn to the question of efficient algorithms for the specific problem of answering queries to classical (i.e., **K**-free) $\mathcal{SROIQ}$ queries. We solve this problem by providing a sound and complete reduction from epistemic querying to standard DL reasoning; our approach reduces occurrences of the **K** operator to intermediate calls to a standard DL reasoner. Employing this technique, existing reasoners for non-epistemic DLs can be reused in a black-box fashion for the task of answering epistemic queries.

Based on this algorithm, we implemented a reasoner capable of answering epistemic queries to OWL ontologies. To this end, we extended the OWL-API (the standard interface for reasoning in OWL) by constructs for epistemic concepts and roles to be used in epistemic queries.

For space reasons, we had to omit most of the numerous and lengthy proofs from the paper. However, we refer the interested reader to the accompanying technical report (Mehdi and Rudolph 2011) where all the technical details are spelled out and full proofs are given.

Preliminaries

We briefly recap the description logic $\mathcal{SROIQ}$ (for details see Horrocks, Kutz, and Sattler 2006) and introduce its extension with the epistemic operator **K**. Let N_I, N_C, and N_R be finite, disjoint sets called *individual names*, *concept names* and *role names* respectively, with N_R being partitioned into *simple* and *non-simple* roles. These atomic entities can be used to form complex ones as displayed in Table 1.

A $\mathcal{SROIQ}$*-knowledge base* is a tuple $(\mathcal{T}, \mathcal{R}, \mathcal{A})$ where $\mathcal{T}$ is a $\mathcal{SROIQ}$-TBox, $\mathcal{R}$ is a regular $\mathcal{SROIQ}$-role hierarchy[1] and $\mathcal{A}$ is a $\mathcal{SROIQ}$-ABox. Table 2 presents the respective axiom types.

[1] We assume the usual regularity assumption for $\mathcal{SROIQ}$, but omit it for space reasons.

Name	Syntax	Semantics
inverse role	R^-	$\{(x,y) \in \Delta^\mathcal{I} \times \Delta^\mathcal{I} \mid (y,x) \in R^\mathcal{I}\}$
universal role	U	$\Delta^\mathcal{I} \times \Delta^\mathcal{I}$
top	$\top$	$\Delta^\mathcal{I}$
bottom	$\bot$	$\emptyset$
negation	$\neg C$	$\Delta^\mathcal{I} \setminus C^\mathcal{I}$
conjunction	$C \sqcap D$	$C^\mathcal{I} \cap D^\mathcal{I}$
disjunction	$C \sqcup D$	$C^\mathcal{I} \cup D^\mathcal{I}$
nominals	$\{a_1, \ldots, a_n\}$	$\{a_1^\mathcal{I}, \ldots, a_n^\mathcal{I}\}$
univ. restriction	$\forall R.C$	$\{x \mid \forall y.(x,y) \in R^\mathcal{I} \to y \in C^\mathcal{I}\}$
exist. restriction	$\exists R.C$	$\{x \mid \exists y.(x,y) \in R^\mathcal{I} \wedge y \in C^\mathcal{I}\}$
Self concept	$\exists S.\mathsf{Self}$	$\{x \mid (x,x) \in S^\mathcal{I}\}$
qualified number	$\leqslant n\, S.C$	$\{x \mid \#\{y \in C^\mathcal{I} \mid (x,y) \in S^\mathcal{I}\} \leq n\}$
restriction	$\geqslant n\, S.C$	$\{x \mid \#\{y \in C^\mathcal{I} \mid (x,y) \in S^\mathcal{I}\} \geq n\}$

Table 1: Syntax and semantics of role and concept constructors in $\mathcal{SROIQ}$. Thereby a denotes an individual name, R an arbitrary role name and S a simple role name. C and D denote concept expressions.

Axiom α	$\mathcal{I} \models \alpha$, if	
$R_1 \circ \cdots \circ R_n \sqsubseteq R$	$R_1^\mathcal{I} \circ \cdots \circ R_n^\mathcal{I} \subseteq R^\mathcal{I}$	RBox $\mathcal{R}$
$\mathsf{Dis}(S,T)$	$S^\mathcal{I} \cap T^\mathcal{I} = \emptyset$	
$C \sqsubseteq D$	$C^\mathcal{I} \subseteq D^\mathcal{I}$	TBox $\mathcal{T}$
$C(a)$	$a^\mathcal{I} \in C^\mathcal{I}$	ABox $\mathcal{A}$
$R(a,b)$	$(a^\mathcal{I}, b^\mathcal{I}) \in R^\mathcal{I}$	
$a \doteq b$	$a^\mathcal{I} = a^\mathcal{I}$	
$a \not\doteq b$	$a^\mathcal{I} \neq b^\mathcal{I}$	

Table 2: Syntax and semantics of $\mathcal{SROIQ}$ axioms

The semantics of $\mathcal{SROIQ}$ is defined via interpretations $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ composed of a non-empty set $\Delta^\mathcal{I}$ called the *domain of* $\mathcal{I}$ and a function $\cdot^\mathcal{I}$ mapping individual names to elements of $\Delta^\mathcal{I}$, concept names to subsets of $\Delta^\mathcal{I}$ and role names to subsets of $\Delta^\mathcal{I} \times \Delta^\mathcal{I}$. This mapping is extended to complex role and concept expressions as in Table 1 and finally used to define satisfaction of axioms (see Table 2). We say that $\mathcal{I}$ satisfies a knowledge base $\Sigma = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ (or $\mathcal{I}$ is a model of Σ, written: $\mathcal{I} \models \Sigma$) if it satisfies all axioms of $\mathcal{T}$, $\mathcal{R}$, and $\mathcal{A}$. We say that a knowledge base Σ *entails* an axiom α (written $\Sigma \models \alpha$) if all models of Σ are models of α.

Furthermore, we let $\mathcal{SROIQK}$ denote the extension of $\mathcal{SROIQ}$ by **K**, where we allow **K** to appear in front of concept or role expressions. We call a $\mathcal{SROIQK}$-role an *epistemic role* if **K** occurs in it. An epistemic role is *simple* if it is of the form **K**S where S is a simple $\mathcal{SROIQ}$-role.

Classical Semantics for Epistemic DLs

Following the way epistemic semantics for DLs have been hitherto defined (see, e.g., Donini et al. 1998), the classical semantics of $\mathcal{SROIQK}$ is given as *possible world semantics* in terms of *epistemic interpretations*. Thereby the following two central assumptions are made:

1. *Common Domain Assumption*: all interpretations are defined over a fixed countably infinite domain Δ.

2. *Rigid Term Assumption*: For all interpretations, the mapping from individuals to domains elements is fixed: it is just the identity function.

Due to these assumptions, we can w.l.o.g. stipulate $\Delta := N_I \cup \mathbb{N}$. Essentially, these assumptions are imposed in order to ensure that (sets of) domain elements can be referred to and dealt with uniformly in a cross-domain manner.

Next, we provide the definition of epistemic interpretations. The main difference to the non-epistemic case, is that we provide a "context" of relevant models which are inspected whenever the extension of an epistemic concept or role is to be determined.

Definition 1 *An* epistemic interpretation *for $\mathcal{SROIQK}$ is a pair $(\mathcal{I}, \mathcal{W})$ where $\mathcal{I}$ is a $\mathcal{SROIQ}$-interpretation and $\mathcal{W}$ is a set of $\mathcal{SROIQ}$-interpretations, where $\mathcal{I}$ and all of $\mathcal{W}$ have the same infinite domain Δ with $N_I \subset \Delta$. The interpretation function $\cdot^{\mathcal{I},\mathcal{W}}$ is then defined as follows:*

$$a^{\mathcal{I},\mathcal{W}} = a \quad \text{for } a \in N_I$$
$$X^{\mathcal{I},\mathcal{W}} = X^{\mathcal{I}} \quad \text{for } X \in N_C \cup N_R \cup \{\top, \bot\}$$
$$\{a_1,..., a_n\}^{\mathcal{I},\mathcal{W}} = \{a_1,..., a_n\}$$
$$(\mathbf{K}C)^{\mathcal{I},\mathcal{W}} = \textstyle\bigcap_{\mathcal{J}\in\mathcal{W}}(C^{\mathcal{J},\mathcal{W}}) \qquad (\mathbf{K}R)^{\mathcal{I},\mathcal{W}} = \textstyle\bigcap_{\mathcal{J}\in\mathcal{W}}(R^{\mathcal{J},\mathcal{W}})$$
$$(C \sqcap D)^{\mathcal{I},\mathcal{W}} = C^{\mathcal{I},\mathcal{W}} \cap D^{\mathcal{I},\mathcal{W}} \qquad (C \sqcup D)^{\mathcal{I},\mathcal{W}} = C^{\mathcal{I},\mathcal{W}} \cup D^{\mathcal{I},\mathcal{W}}$$
$$(\neg C)^{\mathcal{I},\mathcal{W}} = \Delta \setminus C^{\mathcal{I},\mathcal{W}}$$
$$(\exists R.\mathsf{Self})^{\mathcal{I},\mathcal{W}} = \{x \mid (x, x) \in R^{\mathcal{I},\mathcal{W}}\}$$
$$(\exists R.C)^{\mathcal{I},\mathcal{W}} = \{x \mid \exists y.(x, y) \in R^{\mathcal{I},\mathcal{W}} \wedge y \in C^{\mathcal{I},\mathcal{W}}\}$$
$$(\forall R.C)^{\mathcal{I},\mathcal{W}} = \{x \mid \forall y.(x, y) \in R^{\mathcal{I},\mathcal{W}} \rightarrow y \in C^{\mathcal{I},\mathcal{W}}\}$$
$$(\leqslant nR.C)^{\mathcal{I},\mathcal{W}} = \{x \mid \#\{y \in C^{\mathcal{I},\mathcal{W}} \mid (x, y) \in R^{\mathcal{I},\mathcal{W}}\} \leq n\}$$
$$(\geqslant nR.C)^{\mathcal{I},\mathcal{W}} = \{x \mid \#\{y \in C^{\mathcal{I},\mathcal{W}} \mid (x, y) \in R^{\mathcal{I},\mathcal{W}}\} \geq n\}$$

where C and D are $\mathcal{SROIQK}$-concepts and R is a $\mathcal{SROIQK}$-role.

From the above one can see that $\mathbf{K}C$ is interpreted as the set of objects that are in the extension of C under every interpretation in $\mathcal{W}$. This also makes clear why the common domain and rigid term assumption have to be imposed; otherwise the respective extension intersections would be empty. Note that the rigid term assumption implies the unique name assumption (UNA) i.e., for any epistemic interpretation $\mathcal{I} \in \mathcal{W}$ and for any two distinct individual names a and b we have that $a^{\mathcal{I}} \neq b^{\mathcal{I}}$.

The notions of knowledge base, TBox, RBox and Abox, their respective axioms, and their interpretations can be extended from $\mathcal{SROIQ}$ to $\mathcal{SROIQK}$ in the obvious way.

An *epistemic model* for a $\mathcal{SROIQK}$-knowledge base $\Sigma = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ is a *maximal* non-empty set $\mathcal{W}$ of $\mathcal{SROIQ}$-interpretations such that $(\mathcal{I}, \mathcal{W})$ satisfies $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$ for each $\mathcal{I} \in \mathcal{W}$. A $\mathcal{SROIQK}$-knowledge base Σ is said to be *satisfiable* if it has an epistemic model. The knowledge base Σ *(epistemically) entails* an axiom α (written $\Sigma \Vdash \alpha$), if for every epistemic model $\mathcal{W}$ of Σ, we have that for each $\mathcal{I} \in \mathcal{W}$, the epistemic interpretation $(\mathcal{I}, \mathcal{W})$ satisfies α. By definition every $\mathcal{SROIQ}$-knowledge base is a $\mathcal{SROIQK}$-knowledge base. Note that a given $\mathcal{SROIQ}$-knowledge base Σ has up to isomorphism only one unique epistemic model which is the set of all models of Σ having infinite domain and satisfying the unique name assumption. We denote this model by $\mathcal{M}(\Sigma)$.

Problems with the Classical Semantics

Following the intuition that led to the introduction of the $\mathbf{K}$ operator as an extension of $\mathbf{K}$-free standard DL reasoning, a rather intuitive basic requirement to an epistemic DL semantics is arguably the following.

Definition 2 *For a given DL $\mathcal{L}$, an epistemic DL semantics represented by an entailment relation $\Vdash\!\approx$ is called $\mathcal{L}$-*backward-compatible *if it coincides with the (non-epistemic) standard semantics (represented by $\models$) on non-epistemic axioms, i.e. for an $\mathcal{L}$ knowledge base Σ and an $\mathcal{L}$ axiom α both of which not containing $\mathbf{K}$, we have $\Sigma \Vdash\!\approx \alpha$ exactly if $\Sigma \models \alpha$. Moreover, $\Vdash\!\approx$ is called $\mathcal{L}$-*UNA-backward-compatible*, if $\Sigma \Vdash\!\approx \alpha$ exactly if $\Sigma \models \alpha$ under the unique name assumption.*

We can show that $\Vdash$ is $\mathcal{SRIQ}\backslash U$-UNA-backward-compatible, where $\mathcal{SRIQ}\backslash U$ denotes the description logic $\mathcal{SROIQ}$ without nominal concepts and the universal role. The main ingredient for this is the insight that for any finite interpretation of a given $\mathcal{SRIQ}\backslash U$ knowledge base, we can come up with an infinite interpretation such that both interpretations behave in exactly the same way in terms of satisfaction of axioms.

Lemma 1 *Let Σ be a $\mathcal{SRIQ}\backslash U$ knowledge base. For any interpretation $\mathcal{I}$ there is an interpretation $\mathcal{I}_\omega$ with infinite domain such that $\mathcal{I} \models \Sigma$ if and only if $\mathcal{I}_\omega \models \Sigma$.*

Proof sketch. We define $\mathcal{I}_\omega$ as follows:

$$\Delta^{\mathcal{I}_\omega} := \Delta^{\mathcal{I}} \times \mathbb{N},$$
$$a^{\mathcal{I}_\omega} := \langle a^{\mathcal{I}}, 0\rangle \text{ for every } a \in N_I,$$
$$A^{\mathcal{I}_\omega} := \{\langle x, i\rangle \mid x \in A^{\mathcal{I}} \text{ and } i \in \mathbb{N}\} \text{ for each } A \in N_C,$$
$$R^{\mathcal{I}_\omega} := \{(\langle x, i\rangle, \langle x', i\rangle) \mid (x, x') \in R^{\mathcal{I}},\ i \in \mathbb{N}\} \text{ for each } R \in N_R.$$

Obviously, $\mathcal{I}_\omega$ has infinite domain. Coincidence of axiom satisfaction is then proven via structural induction. □

As a consequence, the restriction to infinite models imposed by the common domain assumption turns out to be not essential in that case of $\mathcal{SRIQ}\backslash U$.

However, this situation changes drastically once nominals or the universal role are involved. To see this, consider the axioms $\top \sqsubseteq \{a, b, c\}$ or $\top \sqsubseteq \leqslant 3U.\top$. Each of these axioms considered as a knowledge base Σ has only models with at most three elements. Consequently, in both cases we have that Σ is unsatisfiable w.r.t. the classical epistemic semantics and consequently by *ex falsum quodlibet* we, e.g., obtain $\Sigma \Vdash \top \sqsubseteq \bot$ whereas we clearly have $\Sigma \not\models \top \sqsubseteq \bot$ even under the UNA. So we conclude that $\Vdash$ is not UNA-backward-compatible for any description logic that features nominals or simultaneously number restrictions and the universal role; in particular, it is not $\mathcal{SROIQ}$-UNA-backward-compatible.

While the imposed UNA may be a deliberate decision, we believe that non-$\mathcal{SROIQ}$-UNA-backward-compatibility of classical epistemic entailment is not intended but rather a side effect of a semantics crafted for and probed against less expressive description logics; it contradicts the intuition behind the $\mathbf{K}$ operator. This motivates our quest for a more appropriate, "domain-flexible" epistemic semantics.

A Revised Semantics

In order to allow for the necessary flexibility, we need to relinquish the common domain assumption and the rigid term assumption in the epistemic semantics: The domains we consider in the possible worlds should be allowed to have arbitrary (yet non-empty) size and be composed of arbitrary elements. An individual name may refer to different elements in different possible worlds. Also, individuals denoted by different individual names may coincide in some worlds but not in others.

Still, due to the reasons discussed before, we have to find a substitute for the common domain and rigid term assumptions as otherwise every epistemic role or concept would have the empty set as extension. We solve the problem by introducing one abstraction layer that assigns *abstract individual names* to domain elements. These abstract individual names are elements from $N_I \cup \mathbb{N}$ and hence common to all interpretations, thus they can serve as the "common ground" for different interpretations with different domains. We require that every domain element is associated with at least one abstract name, however, we also allow for different names denoting the same domain element (thus allowing for the possibility of finite domains). This intuition leads to the definition of extended interpretations:

Definition 3 *An* extended $\mathcal{SROIQ}$-interpretation $\mathfrak{I}$ *is a tuple* $(\Delta_\mathfrak{I}, \cdot^\mathfrak{I}, \varphi_\mathfrak{I})$ *such that*

- $(\Delta_\mathfrak{I}, \cdot^\mathfrak{I})$ *is a standard DL interpretation,*
- $\varphi_\mathfrak{I} : N_I \cup \mathbb{N} \twoheadrightarrow \Delta^\mathcal{I}$ *is a surjective function from* $N_I \cup \mathbb{N}$ *onto* $\Delta^\mathcal{I}$*, such that for all* $a \in N_I$ *we have that* $\varphi_\mathfrak{I}(a) = a^\mathfrak{I}$.

Note that the function $\varphi_\mathfrak{I}$ returns the actual interpretation of an individual, given its (abstract) name, under the interpretation $\mathfrak{I}$. We lift $\varphi_\mathfrak{I}$ to sets of names and let $\varphi_\mathfrak{I}^{-1}$ denote the corresponding inverse. Next, we introduce the notion of extended epistemic interpretations.

Definition 4 *(extended semantics for $\mathcal{SROIQK}$) An* extended epistemic interpretation *for $\mathcal{SROIQK}$ is a pair* $(\mathfrak{I}, \mathfrak{W})$*, where $\mathfrak{I}$ is an extended $\mathcal{SROIQ}$-interpretation and $\mathfrak{W}$ is a set of extended $\mathcal{SROIQ}$-interpretations. Similar to epistemic interpretations, we define an extended interpretation function* $\cdot^{\mathfrak{I},\mathfrak{W}}$:

$$
\begin{aligned}
a^{\mathfrak{I},\mathfrak{W}} &= a^\mathfrak{I} \quad \textit{for } a \in N_I\\
X^{\mathfrak{I},\mathfrak{W}} &= X^\mathfrak{I} \quad \textit{for } X \in N_C \cup N_R \cup \{\top, \bot\}\\
\{a_1,..., a_n\}^{\mathfrak{I},\mathfrak{W}} &= \{a_1^\mathfrak{I},..., a_n^\mathfrak{I}\}\\
(C \sqcap D)^{\mathfrak{I},\mathfrak{W}} &= C^{\mathfrak{I},\mathfrak{W}} \cap D^{\mathfrak{I},\mathfrak{W}}\\
(C \sqcup D)^{\mathfrak{I},\mathfrak{W}} &= C^{\mathfrak{I},\mathfrak{W}} \cup D^{\mathfrak{I},\mathfrak{W}}\\
(\neg C)^{\mathfrak{I},\mathfrak{W}} &= \Delta^\mathfrak{I} \setminus C^{\mathfrak{I},\mathfrak{W}}\\
(\exists R.\mathsf{Self})^{\mathfrak{I},\mathfrak{W}} &= \{x \mid (x, x) \in R^{\mathfrak{I},\mathfrak{W}}\}\\
(\forall R.C)^{\mathfrak{I},\mathfrak{W}} &= \{x \mid \forall y.(x, y) \in R^{\mathfrak{I},\mathfrak{W}} \rightarrow y \in C^{\mathfrak{I},\mathfrak{W}}\}\\
(\exists R.C)^{\mathfrak{I},\mathfrak{W}} &= \{x \mid \exists y.(x, y) \in R^{\mathfrak{I},\mathfrak{W}} \wedge y \in C^{\mathfrak{I},\mathfrak{W}}\}\\
(\leqslant nR.C)^{\mathfrak{I},\mathfrak{W}} &= \{x \mid \#\{y \in C^{\mathfrak{I},\mathfrak{W}} \mid (x, y) \in R^{\mathfrak{I},\mathfrak{W}}\} \leq n\}\\
(\geqslant nR.C)^{\mathfrak{I},\mathfrak{W}} &= \{x \mid \#\{y \in C^{\mathfrak{I},\mathfrak{W}} \mid (x, y) \in R^{\mathfrak{I},\mathfrak{W}}\} \geq n\}\\
(\mathbf{K}C)^{\mathfrak{I},\mathfrak{W}} &= \varphi_\mathfrak{I}\left(\textstyle\bigcap_{\mathfrak{J}\in\mathfrak{W}} \varphi_\mathfrak{J}^{-1}\left(C^{\mathfrak{J},\mathfrak{W}}\right)\right)\\
(\mathbf{K}R)^{\mathfrak{I},\mathfrak{W}} &= \varphi_\mathfrak{I}\left(\textstyle\bigcap_{\mathfrak{J}\in\mathfrak{W}} \varphi_\mathfrak{J}^{-1}\left(R^{\mathfrak{J},\mathfrak{W}}\right)\right)
\end{aligned}
$$

Again, we set $[(\mathbf{K}R)^-]^{\mathfrak{I},\mathfrak{W}} := (\mathbf{K}R^-)^{\mathfrak{I},\mathfrak{W}}$ *for an epistemic role* $(\mathbf{K}R)^-$.

The semantics of ABox, TBox, RBox and ABox axioms follows exactly that for the classical semantics. Here, instead of $\models\!\!\models$, we use the symbol $\models\!\!\models_e$, where e indicates that the relation is w.r.t. the extended semantics.

An *extended epistemic model* for a $\mathcal{SROIQK}$-knowledge base $\Sigma = (\mathcal{T}, \mathcal{R}, \mathcal{A})$ is a *maximal* non-empty set $\mathcal{W}$ of extended $\mathcal{SROIQ}$-interpretations such that $(\mathcal{I}, \mathcal{W})$ satisfies $\mathcal{T}$, $\mathcal{R}$ and $\mathcal{A}$ for each $\mathcal{I} \in \mathcal{W}$. A $\mathcal{SROIQK}$-knowledge base Σ is *satisfiable* (under the extended semantics) if it has an extended epistemic model. Similarly, the knowledge base Σ *entails* an axiom α under the extended semantics, written $\Sigma \models\!\!\models_e \alpha$, if for every extended epistemic model $\mathcal{W}$ of Σ, we have that for every $\mathcal{I} \in \mathcal{W}$, the extended epistemic interpretation $(\mathcal{I}, \mathcal{W})$ satisfies α.

We now first note that the newly established semantics has the desired compatibility property.

Theorem 1 $\models\!\!\models_e$ *is $\mathcal{SROIQ}$-backward-compatible.*

Proof sketch: First note that every satisfiable $\mathbf{K}$-free knowledge base Σ has exactly one extended epistemic model

$$\mathfrak{M}(\Sigma) = \{(\Delta_\mathfrak{I}, \cdot^\mathfrak{I}, \varphi_\mathfrak{I}) \mid (\Delta_\mathfrak{I}, \cdot^\mathfrak{I}) \models \Sigma,\ \varphi_\mathfrak{I} = \cdot^\mathfrak{I}|_{N_I} \cup f, f{:}\mathbb{N} \twoheadrightarrow \Delta_\mathfrak{I}\}.$$

Hence we have $\Sigma \models\!\!\models_e \alpha$ exactly if every $\mathfrak{I} \in \mathcal{M}_e(\Sigma)$ satisfies α, which (presuming α being $\mathbf{K}$-free) is the case exactly if $\Sigma \models \alpha$. □

Consequently, this new semantics is more adequate for very expressive DLs such as $\mathcal{SROIQ}$. Yet, as will be shown later in the paper, it is also generic in the sense that for $\mathcal{SRIQ} \backslash \mathcal{U}$ knowledge bases it behaves similar to the (classical) epistemic interpretation introduced earlier. With this new semantics, we avoid the aforementioned problems arising from nominals and the universal role in the language of a knowledge base. Arguably, this makes the revisited semantics a more suitable and appropriate choice for $\mathbf{K}$-extensions of expressive description logics, like $\mathcal{SROIQK}$.

Reducing Epistemic Querying to Standard DL Reasoning

We next introduce a novel technique for answering epistemic queries to $\mathcal{SROIQ}$ knowledge bases under the revised semantics. More precisely, we provide a way of checking whether a given knowledge base entails concept assertions, role assertions or concept subsumptions where the involved concepts and roles may contain $\mathbf{K}$. Our method reduces this problem to a number of iterative entailment checks for $\mathbf{K}$-free axioms. To justify the translation, we establish two lemmata that characterize possible instances of epistemics concepts and roles, respectively.

Lemma 2 *Let Σ be a $\mathcal{SROIQ}$-knowledge base and $C = \mathbf{K}D$ an epistemic concept where D is $\mathbf{K}$-free. For an extended interpretation $\mathfrak{I} \in \mathfrak{M}(\Sigma)$ and $x \in \Delta^\mathfrak{I}$, we have that $x \in C^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ exactly if one of the following is the case:*

1. $\Sigma \models \top \sqsubseteq D$*, or*
2. $x = a^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ *and* $\Sigma \models D(a)$ *for an individual name* $a \in N_I$.

Intuitively, this lemma ensures that the extension of a concept that is preceded by **K** can only contain named individuals unless it comprises the whole domain. For roles we get a more intricate case distinction, however, it boils down to characterizing the set of "(inverse) role neighbors" of a fixed individual as the whole domain or a set of named individuals. As an "exceptional case" to this, we might get the diagonal of $\Delta^{\mathfrak{I}} \times \Delta^{\mathfrak{I}}$ as additional instances of an epistemic role.

Lemma 3 *Let Σ be a $\mathcal{SROIQ}$-knowledge base. Let $R = \mathbf{K}P$ be an epistemic role. For any extended interpretation $\mathfrak{I} \in \mathfrak{M}(\Sigma)$ and any $x, y \in \Delta^{\mathfrak{I}}$, we have that $(x, y) \in R^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ exactly if one of the following holds:*

1. *$\Sigma \models U \sqsubseteq P$, or*
2. *$x = a^{\mathfrak{I},\mathfrak{M}(\Sigma)}$, $y = b^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ and $\Sigma \models P(a, b)$ for some individual names $a, b \in N_I$, or*
3. *$x = a^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ and $\Sigma \models \top \sqsubseteq \exists P^{-}.\{a\}$ for some individual name $a \in N_I$, or*
4. *$y = b^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ and $\Sigma \models \top \sqsubseteq \exists P.\{b\}$ for some individual name $b \in N_I$, or*
5. *$x = y$ and $\Sigma \models \top \sqsubseteq \exists P.\mathsf{Self}$.*

These two preceding lemmas now give rise to a translation of epistemic concept expressions into equivalent **K**-free ones. Note that the translation itself requires to check entailment of (**K**-free) axioms, hence it is not strictly syntactical and it depends on the underlying knowledge base.

Definition 5 *(Translation Function $[\![\cdot]\!]_\Sigma$) Let Σ be a $\mathcal{SROIQ}$-knowledge base. For a $\mathcal{SROIQ}$ concept A and a $\mathcal{SROIQ}$ role R, let $\mathrm{trg}_\Sigma^{A,R}$ denote the nominal concept $\{a_1, \ldots, a_n\}$ containing all a_i for which $\Sigma \models A \sqsubseteq \exists R.\{a_i\}$ and let $\mathrm{trg}_\Sigma^{A,R} = \bot$ if there are no such a_i. We recursively define the function $[\![\cdot]\!]_\Sigma$ mapping $\mathcal{SROIQK}$ concept expressions to $\mathcal{SROIQ}$ concept expressions:*

$$[\![C]\!]_\Sigma = C \quad \text{if } C \text{ is from } N_I \cup \{\top, \bot\}, \text{ a nominal, or a } \mathbf{K}\text{-free self concept;}$$

$$[\![C_1 \sqcap C_2]\!]_\Sigma = [\![C_1]\!]_\Sigma \sqcap [\![C_2]\!]_\Sigma$$

$$[\![C_1 \sqcup C_2]\!]_\Sigma = [\![C_1]\!]_\Sigma \sqcup [\![C_2]\!]_\Sigma$$

$$[\![\neg C]\!]_\Sigma = \neg [\![C]\!]_\Sigma$$

$$[\![\Xi R.D]\!]_\Sigma = \Xi R.[\![D]\!]_\Sigma \quad \text{for } \Xi \in \{\forall, \exists, \geqslant n, \leqslant n\},\ R\ \mathbf{K}\text{-free}$$

$$[\![\mathbf{K}D]\!]_\Sigma = \begin{cases} \top & \text{if } \Sigma \models [\![D]\!]_\Sigma \equiv \top \\ \{a \in N_I \mid \Sigma \models [\![D]\!]_\Sigma(a)\} & \text{otherwise} \end{cases}$$

$$[\![\exists \mathbf{K}S.\mathsf{Self}]\!]_\Sigma = [\![\mathbf{K}\exists S.\mathsf{Self}]\!]_\Sigma$$

$$[\![\Xi \mathbf{K}R.D]\!]_\Sigma = \Xi R.[\![D]\!]_\Sigma \quad \text{for } \Xi \in \{\forall, \exists, \geqslant n, \leqslant n\} \text{ and } \Sigma \models R \equiv U$$

$$[\![\forall \mathbf{K}P.D]\!]_\Sigma = \neg [\![\exists \mathbf{K}P.\neg D]\!]_\Sigma$$

$$[\![\exists \mathbf{K}P.D]\!]_\Sigma = \exists P.(\mathrm{trg}_\Sigma^{\top,P} \sqcap [\![D]\!]_\Sigma) \sqcup (\mathrm{trg}_\Sigma^{\top,P^-} \sqcap \exists P.[\![D]\!]_\Sigma) \sqcup \bigsqcup_{a \in N_I} (\{a\} \sqcap \exists P.(\mathrm{trg}_\Sigma^{\{a\},P} \sqcap [\![D]\!]_\Sigma)) \underbrace{\sqcup [\![D]\!]_\Sigma}_{\text{only if } \Sigma \models \top \sqsubseteq \exists P.\mathsf{Self}}$$

$$[\![\leqslant n \mathbf{K}P.D]\!]_\Sigma = \neg [\![\geqslant (n{+}1) \mathbf{K}P.D]\!]_\Sigma$$

$$[\![\geqslant n \mathbf{K}P.D]\!]_\Sigma = \geqslant nP.(\mathrm{trg}_\Sigma^{\top,P} \sqcap [\![D]\!]_\Sigma) \sqcup (\mathrm{trg}_\Sigma^{\top,P^-} \sqcap \geqslant nP.[\![D]\!]_\Sigma) \sqcup \bigsqcup_{a \in N_I} (\{a\} \sqcap \geqslant nP.(\mathrm{trg}_\Sigma^{\{a\},P} \sqcap [\![D]\!]_\Sigma\})) \underbrace{\sqcup (\neg\{a \mid a \in N_I\} \sqcap [\![D]\!]_\Sigma \sqcap \geqslant (n{-}1)P.(\mathrm{trg}_\Sigma^{\top,P} \sqcap [\![D]\!]_\Sigma))}_{\text{only if } \Sigma \models \top \sqsubseteq \exists P.\mathsf{Self}}$$

Observe that by definition, the result of applying this function to an epistemic concept indeed yields a concept containing no **K**. Moreover the following lemma, which can be proven by structural induction over the concept expression, ensures that the translation function preserves the concept extension.

Lemma 4 *Let Σ be a $\mathcal{SROIQ}$-knowledge base and C be a $\mathcal{SROIQK}$ concept. Then for any extended interpretation $\mathfrak{I} \in \mathfrak{M}(\Sigma)$, we have that $C^{\mathfrak{I},\mathfrak{M}(\Sigma)} = [\![C]\!]_\Sigma{}^{\mathfrak{I},\mathfrak{M}(\Sigma)}$.*

Consequently this lemma can be employed to prove our main result justifying our approach of deciding entailment of epistemic axioms based on non-epistemic standard reasoning.

Theorem 2 *For a $\mathcal{SROIQ}$-knowledge base Σ, $\mathcal{SROIQK}$ concept C, D, and an individual a, the following hold:*

1. *$\Sigma \models_e C(a)$ if and only if $\Sigma \models [\![C]\!]_\Sigma(a)$.*
2. *$\Sigma \models_e C \sqsubseteq D$ if and only if $\Sigma \models [\![C]\!]_\Sigma \sqsubseteq [\![D]\!]_\Sigma$.*

Proof sketch: For the first case, note that $\Sigma \models_e C(a)$ is equivalent to $a^{\mathfrak{I},\mathfrak{M}(\Sigma)} \in C^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ which by Lemma 4 implies that $a^{\mathfrak{I},\mathfrak{M}(\Sigma)} \in [\![C]\!]_\Sigma{}^{\mathfrak{I},\mathfrak{M}(\Sigma)}$ for all $\mathfrak{I} \in \mathfrak{M}(\Sigma)$. Since Σ and $[\![C]\!]_\Sigma$ are **K**-free, the correspondence follows from $\mathcal{SROIQ}$-backwards-compatibility as established in Theorem 1. The second case is proven in the same way. □

Finally, we are also able to establish the correspondence that the classical and the newly introduced semantics coincide, as far as epistemic querying on $\mathcal{SRIQ}\backslash U$ knowledge bases is concerned. This result further substantiates our claim that our semantics is a natural extension of the original intuition behind epistemic DLs.

Theorem 3 *Let Σ be a $\mathcal{SRIQ}\backslash U$ knowledge base, C and D $\mathcal{SROIQK}$ concepts, and a an individual name. Then, the following hold:*

1. *$\Sigma \models_e C(a)$ under the unique name assumption if and only if $\Sigma \Vdash C(a)$.*
2. *$\Sigma \models_e C \sqsubseteq D$ under the unique name assumption if and only if $\Sigma \Vdash C \sqsubseteq D$.*

This can be proven by providing a transformation function similar to $[\![\cdot]\!]_\Sigma$ for the classical semantics, proving its correctness and showing that it coincides with $[\![\cdot]\!]_\Sigma$ on $\mathcal{SRIQ}\backslash U$ knowledge bases.

Implementation

Based on the results established in the preceding section, we have implemented a preliminary prototype for epistemic querying of OWL ontologies. In this section, we discuss a few implementation aspects and provide some observations made during such experiments.

The system takes an epistemic concept as input and translates it into an equivalent non-epistemic one according to Definition 5. As argued earlier, several calls to an underlying standard OWL reasoner are involved in this process. The implementation is done in Java on top of the OWL-API.[2] As

[2] http://owlapi-sourceforge.net

the standard OWL-API does not support the epistemic operator, we extended its classes and interfaces with the constructs for epistemic concepts and roles – OWLObjectEpistemicConcept and OWLObjectEpistemicRole. These new classes are derived from the respective standard types OWLBooleanClassExpression and OWLObjectPropertyExpression to fit the design of the OWL-API. A running implemented system has been shared on googlecode[3] and can be downloaded for testing.

First tests on the Wine ontology[4] show that epistemic querying based on our method is feasible in principle also if heavy-weight axiomatization is involved. In case of tests performed on the original Wine ontology, runtimes were about one to two orders of magnitude higher than for the same concept with **K**s removed (note however, that a direct comparison is debatable as the semantics of the two concepts differ). Still, performance degrades strongly if large sets of individuals are involved. This can be explained from the fact that the size of the intermediate concepts generated by the translation increases with the number of ABox individuals. Besides we found that the position of **K** in an epistemic concept also affects the overall computation time. For example, we observed that it takes more time to translate a concept where **K** is preceded by a negation.

Conclusion and Outlook

We showed that some expressive features of today's DLs such as $\mathcal{SROIQ}$ cause problems when applying the hitherto used semantics to epistemically extended DLs. We suggested a revision to the semantics and proved that this revised semantics solves the aforementioned problem while coinciding with the traditional semantics on less expressive DLs (up to $\mathcal{SRIQ}\backslash U$). Focusing on the new semantics, we provided a way of answering epistemic queries to $\mathcal{SROIQ}$ knowledge bases via a reduction to a series of standard reasoning steps, thereby enabling the deployment of the available highly optimized off-the-shelf DL reasoners. Finally, we presented an implementation allowing for epistemic querying in OWL 2 DL.

Avenues for future research include the following: First, we will investigate to what extent the methods described here can be employed for entailment checks on $\mathcal{SROIQK}$ knowledge bases, i.e., in cases where **K** occurs inside the knowledge base. In that case, stronger non-monotonic effects occur and the unique-epistemic-model property is generally lost. On the more practical side, we aim at further developing our initial prototype. We are confident that by applying appropriate optimizations such as caching strategies and syntactic query preprocessing a significant improvement in terms of runtime can be achieved. Moreover, we intend to perform extensive tests with different available OWL reasoners; in our case an efficient handling of (possibly rather extensive) nominal concepts is crucial for a satisfactory performance. In the long run, we aim at demonstrating the added value of epistemic querying by providing an appropriate user-front-end and performing user studies. Furthermore, we will propose an extension of the current OWL standard by epistemic constructs in order to provide a common ground for future applications. Finally, we will study the correspondance between our sematnics and the one provided in (Motik and Rosati 2010) which is is based on the standard name assumption (SNA).

[3]`http://code.google.com/p/epistemicdl/`

[4]`http://www.w3.org/TR/owl-guide/wine.rdf`

Acknowledgments

This work is supported by the German Federal Ministry of Education and Research (BMBF) under the SAW-Project NanON.

References

Baader, F.; Calvanese, D.; McGuinness, D.; Nardi, D.; and Patel-Schneider, P. F., eds. 2003. *The Description Logic Handbook: Theory, Implementation, and Applications*. Cambridge University Press.

Braüner, T., and Ghilardi, S. 2006. First-order modal logic. In Blackburn, P.; Wolter, F.; and van Benthem, J., eds., *Handbook of Modal Logic*. Elsevier. 549–620.

Donini, F. M.; Lenzerini, M.; Nardi, D.; Schaerf, A.; and Nutt, W. 1992. Adding epistemic operators to concept languages. In Bernhard Nebel, Charles Rich, W. R. S., ed., *Proceedings of the 3rd International Conference on Principles of Knowledge Representation and Reasoning (KR'92)*, 342–353. Morgan Kaufmann.

Donini, F. M.; Lenzerini, M.; Nardi, D.; Schaerf, A.; and Nutt, W. 1998. An epistemic operator for description logics. *Artificial Intelligence* 100(1-2):225–274.

Donini, F.; Nardi, D.; and Rosati, R. 1995. Non-first-order features in concept languages. In *Proceedings of the Fourth Conference of the Italian Association for Artificial Intelligence (AI*IA'95)*, 91–102. Springer.

Donini, F. M.; Nardi, D.; and Rosati, R. 1997. Autoepistemic description logics. In *Proceedings of the Fifteenth International Joint Conference on Artificial Intelligence (IJCAI'97)*, 136–141. Morgan Kaufmann.

Fitting, M., and Mendelsohn, R. 1998. *First-Order Modal Logic*. Kluwer Academic Publishers.

Hitzler, P.; Krötzsch, M.; and Rudolph, S. 2009. *Foundations of Semantic Web Technologies*. Chapman & Hall/CRC.

Horrocks, I.; Kutz, O.; and Sattler, U. 2006. The even more irresistible $\mathcal{SROIQ}$. In Doherty, P.; Mylopoulos, J.; and Welty, C. A., eds., *Proceedings of the 10th International Conference on Principles of Knowledge Representation and Reasoning (KR'06)*, 57–67. AAAI Press.

Levesque, H. J. 1984. Foundations of a functional approach to knowledge representation. *Artificial Intelligence* 23(2):155–212.

Mehdi, A., and Rudolph, S. 2011. Revisiting semantics for epistemic extensions of description logics. Techreport, Institute AIFB, KIT, Karlsruhe. available at `http://www.aifb.kit.edu/images/b/b1/ExtendedSemantics.pdf`.

Motik, B., and Rosati, R. 2010. Reconciling description logics and rules. *J. ACM* 57(5).

OWL Working Group, W. 2009. *OWL 2 Web Ontology Language: Document Overview*. W3C Recommendation. Available at http://www.w3.org/TR/owl2-overview/.

Reiter, R. 1992. What should a database know? *Journal of Logic Programming* 14(1-2):127–153.

Schmidt-Schauß, M., and Smolka, G. 1991. Attributive concept descriptions with complements. *Artificial Intelligence* 48(1):1–26.